**Linear Independence and Spanning**

**Linear Dependence and Independence**

A [set](https://en.wikipedia.org/wiki/Set_(mathematics)) of [vectors](https://en.wikipedia.org/wiki/Vector_(mathematics)) is said to be **linearly dependent** if at least one of the vectors in the set can be defined as a [linear combination](https://en.wikipedia.org/wiki/Linear_combination) of the others; if no vector in the set can be written in this way, then the vectors are said to be **linearly independent**.

**Definition**

If is a non-empty set of vectors in a vector space V, then the vector equation

has at least one solution, namely,

We call this the trivial solution. If this is the only solution, then S is said to be linearly independent set. If there are solutions in addition to the trivial solution, then S is called as linearly dependent set.

**Example 1:** Determine whether the set consists of vectors are linearly dependent or independent.

**Solution:** Let be the linear combination. Then,

From equation (2), we get

Putting value of in equation (1), we get:

Let therefore

As,

So, the vectors are linearly dependent.

**Example 2:** Determine whether the set consists of vectors are linearly dependent or independent.

**Solution:**

Let

From (1), we get,

From (2), we get,

Put values of and in equation (3), we get:

Vectors are linearly independent.

**Exercise:** Determine whether vectors are linearly dependent or independent in .

**Exercise:** Does form a linearly independent set of vectors in

**Example 3:** Are the vectors

in are linearly independent?

**Solution:** Let

,

Hence the vectors are linearly independent.

**Example 4:** Are the vectors in are linearly independent?

**Solution:** Let

By using Gaussian elimination, it is a trivial exercise to show that these vectors are linearly independent.

**Spanning Set**

Let S be the set of some vectors of vector space V. If every vector in V is a linear combination of vectors in S, then the set S is said to span V or the V is spanned by the set S.

**DEFINITION 3**

The subspace of a vector space *V* that is formed from all possible linear combinations of the vectors in a nonempty set *S* is called the ***span of S***, and we say that the vectors in ***S span*** that subspace. If , then we denote the span of *S* by

Example1: Let be the set of standard unit vectors in. Prove that S spans

Solution:

S spans means every vector of can be written as linear combination of these three vectors i.e.

, ,

e.g.

Note: The Standard Unit Vectors Span *Rn*. How?

Recall that the standard unit vectors in are

These vectors span since every vector in can be expressed as

which is a linear combination of . Thus, as in example 1, the vectors

span since every vector in this space can be expressed as linear combination of these three vectors (also called basis).

Can there be vectors other than standard unit vectors in Rn which can span Rn?

Example2: Let V be the vector spaceand . Check span.

Solution: To check whether span we pick any vector in and determine whether there are constants such that

From equation (2)

From equation (3)

Put the values in (1)

Put in (5)

Put in (4)

Thus span, i.e.

Example3: Determine whether span.

Solution: Let

From equation (2)

Subtract (1) and (3)

Put in above equation,

Not possible, No solution.

So do not span.

Another Method: For the matrix of coefficients

So the system has no solution.

System is consistent if & only if its coefficient matrix has non-zero determinant i.e..

**Example 4:** Show that the set

span the vector space

**Solution:** For spanning, let

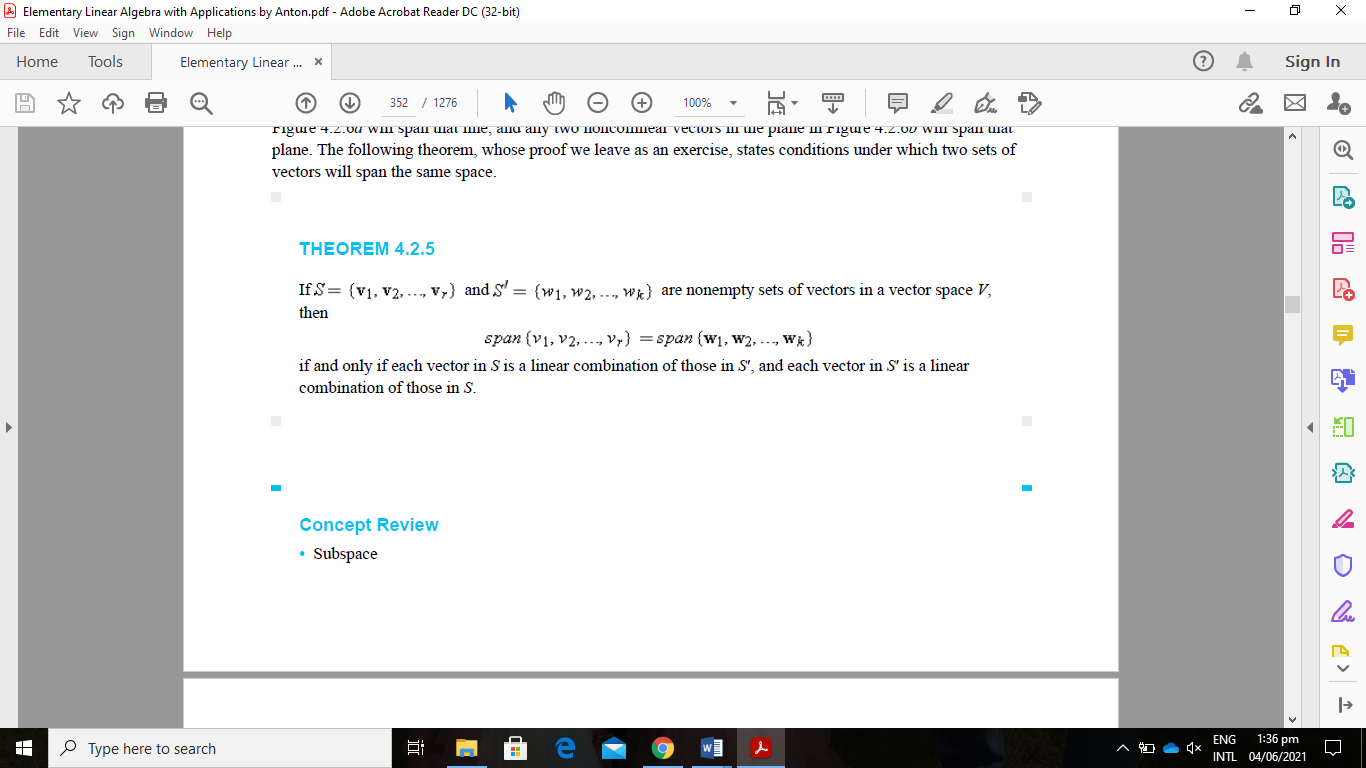
Put in equation (3)

Add (2) and (4)

And by subtracting both implies

So, .

It means S spans V.



**THEOREM 4.3.3**

Let , be a set of vectors in . If , then *S* is linearly dependent.

**EXAMPLE 5.** An Important Linearly Independent Set in *Pn* .

Show that the polynomials

form a linearly independent set in *Pn*.

**Exercise 4.2**

**Q11:** In each part determine whether the given vector span.

Answer: (a) and (d) span.

**Q2:** Let V be the vector space. Let.

Does span V?

**Q3:** In let and Determine whether the vector

Belong to the span.

**Exercise 4.3**

